Discovering Math
Concepts in Business Mathematics
Economics and Finance
Teacher's Guide

Program Description

This program shows how algebraic functions are used in business and finance to calculate and compare rates of simple and compound interest, to efficiently store products, and to track profit and loss in a business setting.

Onscreen Questions

Part 1, "Percent and Principal: Simple Interest," "Money in the Bank: Compound Interest," "Linear Programming: Business Constraints," "Making Dough: Profit and Loss"

- What is the difference between simple and compound interest?

- What are some expenses or costs of running a restaurant?

Part 2, "Math and Mercantilism in the Colonial Era"

- Why was the *Treviso Arithmetic* an important book?

- What were some advantages of using money over the barter system?

Lesson Plan

Student Objectives

- Research the history of money and share presentations.

- Solve problems of proportions using the Rule of Three and the Inverse Rule of Three.

- Convert American and foreign currencies.

- Calculate problems of limiting value using the formula $S = Pe^{rt}$ as it pertains to annual percentage rate.

Materials

- *Concepts in Business Mathematics: Economics and Finance* video

- Computer with Internet access

- Print resources about the history of money

Procedures

1. Have students research the history of money using print and Web resources. The following Web sites are a good starting point:

   - "Origins of Money and Banking
     http://www.ex.ac.uk/~RDavies/arian/origins.html

   - NOVA Online / The History of Money
     http://www.pbs.org/wgbh/nova/moolah/history.html

   - Money
     http://en.wikipedia.org/wiki/Money#History_of_money

2. When students have completed their research, ask them to summarize their findings in a one-page report.

3. Have each student choose a partner. Ask students to share their reports with their partners and answer any questions. Then have students summarize their partners' reports for the class, including at least three interesting facts.

4. Show students examples of proportions and solve them using the Rule of Three. Ask students how the terms in the proportion correspond to the terms in the statement of the Rule of Three. Allow students time to practice.

5. Show students examples of proportions and solve them using the Inverse Rule of Three. Ask students how the terms in the proportion correspond to the terms in the statement of the Inverse Rule of Three. Allow students time to practice.

6. Have students look up current exchange rates. Good resources include these:

   - FXConverter 164 Currency Converter
     http://www.oanda.com/convert/classic

   - Currency Converter
     http://finance.yahoo.com/currency?u

   Have students choose an amount of American dollars and convert it to three different currencies. Also have them explain how to find the exchange rate from dollars to pesos if they are given the exchange rate from pesos to dollars.

7. Using a fixed yearly interest rate, provide students with several examples of total interest and annual percentage rate when varying the number of compounding intervals. Allow students time to practice.

8. Show students an example of a limiting value, for example of the series $1, 1 + \frac{1}{2}, 1 + \frac{1}{2} + \frac{1}{4},$
    and so on. (The series has a limiting value of 2 because each number in the series is halfway between the previous number and 2.) Explain that similarly, *e* is a limiting value that arises in the calculation of annual percentage rate. Allow students time to practice with the formula $S = Pe^{rt}$.

Assessment

Use the following three-point rubric to evaluate students' work during this lesson.

- **3 points:** Students thoroughly researched the history of money and shared an accurate presentation; correctly solved all problems of proportions; accurately converted currencies; and correctly solved all problems of limiting value.

- **2 points:** Students adequately researched the history of money and shared a mostly accurate presentation; correctly solved most of the problems of proportions; accurately converted most currencies; and correctly solved most problems of limiting value.

- **1 point:** Students poorly researched the history of money and shared a mostly inaccurate presentation; incorrectly solved all or most problems of proportions; inaccurately converted most or all currencies; and incorrectly solved all or most problems of limiting value.

Vocabulary

barter
Definition: A mercantile system by which goods and services are directly traded for each other and no formal currency is present
Context: Developing societies often improve their economic efficiency by introducing currency over a barter system.

compound interest
Definition: Interest that accumulates on both the initial amount of the loan (the principal) and the interest already accumulated
Context: Most banks use compound interest instead of simple interest.

cost
Definition: Expenses incurred, as by a business
Context: If a business makes $1,000 and spends $750 for a difference of $250, its costs total $750.

exchange rate
Definition: The factor by which the amount of one currency is multiplied to yield an equivalent amount of another currency

Context: If, at a given time, 100 U.S dollars are worth as much as 110 Canadian dollars, 1.1 Canadian dollars to U.S. dollars is the exchange rate.

interest

Definition: Money generated by principal over time, as in a loan or savings account

Context: If you borrow $500 and agree to pay it back with 5% of the $500 added in a year, the extra 5% is the interest and 5% is said to be the interest rate.

principal

Definition: The initial amount of the loan, before interest is added or paid

Context: If you take out a loan in the amount of $250 that you will pay back with interest, the $250 is the principal.

Academic Standards

National Council of Teachers of Mathematics (NCTM)

The National Council of Teachers of Mathematics provides guidelines for teaching mathematics in grades K–12 to promote mathematical literacy. To view the standards, visit this Web site: http://standards.nctm.org/document/chapter3/index.htm

This lesson plan addresses the following thematic standards:

- Solve problems that arise in mathematics and other contexts

- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole

- Recognize and apply mathematics in contexts outside of mathematics

Mid-continent Research for Education and Learning (McREL)

McREL's Content Knowledge: A Compendium of Standards and Benchmarks for K–12 Education addresses 14 content areas. To view the standards and benchmarks, visit http://www.mcrel.org/compendium/browse.asp .

This lesson plan addresses the following national standards:

- Mathematics: Understands and applies basic and advanced properties of the concepts of numbers; understands and applies basic and advanced properties of functions and algebra; understands the general nature and uses of mathematics

- Economics: Understands savings, investment, and interest rates

- History: United States History: Understands how political, religious, and social institutions emerged in the English colonies

Support Materials

Develop custom worksheets, educational puzzles, online quizzes, and more with the free teaching tools offered on the Discoveryschool.com Web site. Create and print support materials, or save them to a Custom Classroom account for future use. To learn more, visit

- http://school.discovery.com/teachingtools/teachingtools.html

DVD Content

This program is available in an interactive DVD format. The following information and activities are specific to the DVD version.

How To Use the DVD

The DVD starting screen has the following options:

Play Video—This plays the video from start to finish. There are no programmed stops, except by using a remote control. With a computer, depending on the particular software player, a pause button is included with the other video controls.

Video Index—Here the video is divided into sections indicated by video thumbnail icons; brief descriptions are noted for each one. Watching all parts in sequence is similar to watching the video from start to finish. To play a particular segment, press Enter on the remote for TV playback; on a computer, click once to highlight a thumbnail and read the accompanying text description and click again to start the video.

Curriculum Units—These are specially edited video segments pulled from different sections of the video (see below). These nonlinear segments align with key ideas in the unit of instruction. They include onscreen pre- and post-viewing questions, reproduced below in this Teacher's Guide. Total running times for these segments are noted. To play a particular segment, press Enter on the TV remote or click once on the Curriculum Unit title on a computer.

Standards Link—Selecting this option displays a single screen that lists the national academic standards the video addresses.

Teacher Resources—This screen gives the technical support number and Web site address.

Video Index

I. Percent and Principal: Simple Interest (5 min.)
Calculate simple interest given the principal, the interest rate, and the time. Then calculate the interest rate given the principal, the end value, and the time.

II. Money in the Bank: Compound Interest (5 min.)
Learn how compound and simple interest compare. Compute compound interest given the principal, the interest rate, and the time.

III. Linear Programming: Business Constraints (3 min.)
Discover the idea of maximizing profit within a set of business constraints. Learn to represent constraints graphically.

IV. Making Dough: Profit and Loss (6 min.)
Explore the concepts of cost, revenue, and profit and know how to calculate them, as well as kinds of decisions businesses make. Learn the significance of break-even points and how to calculate them.

V. Math and Mercantilism in the Colonial Era (27 min.)
Explore the concepts of currency, tare, tret, barter, and interest. Appreciate the ways in which these concepts evolved and influenced history, from Renaissance Venice to Colonial America.

Curriculum Units

1. Simple Interest
Pre-viewing question
Q. Name a benefit and a drawback to taking out a loan.
A. Answers will vary.

Post-viewing question
Q. If you borrow $100 for six years at an interest rate of 5% per year, how much simple interest will you pay?
A. The amount of interest is given by the formula $I=prt$; in this case, $I = (100)(.05)(6) = 30$, which is $30 of simple interest.

2. Compound Interest
Pre-viewing question
Q. How do you calculate simple interest?
A. Multiply the principal, the interest rate, and the time of the loan.

Post-viewing question
Q. What is the formula for compound interest? Define all the variables.

A. The formula for compound interest is $I = P\left(1+\dfrac{r}{n}\right)^{nt}$; I is the interest, P is the principal, r is the yearly interest rate, n is the number of compounding intervals per year, and t is the t years.

3. Business Constraints
Pre-viewing question
Q. Name three factors that might influence the way a store purchases merchandise.
A. Answers will vary.

Post-viewing question
Q. In linear programming, the profit-maximizing point on a polygon resulting from the constraints is found at what kind of point of the polygon?
A. A vertex.

4. Profit and Loss
Pre-viewing question
Q. What makes a business successful?
A. Answers will vary.

Post-viewing question
Q. What is the break-even point and how do you find it?
A. The break-even point is the point at which a business neither loses money nor profits. It is found by setting formulas for cost and revenue equal to each other, or by graphing, where the break-even point will be the intersection of the cost function and the revenue function.

5. The Rule of Three
Pre-viewing question
Q. What did Renaissance Italy contribute to modern culture?
A. Answers will vary.

Post-viewing question
Q. What does the Rule of Three state?
A. If a is to b as c is to d, and a, b, and c are known, then d can be found and is equal to c times b divided by a.

6. The Inverse Rule of Three
Pre-viewing question
Q. It takes x men y hours to do a job. What happens to the time if the number of men is increased or decreased?
A. If the number of men is increased, the time (y) decreases. If the number of men is decreased, the time increases.

Post-viewing question
Q. What is the difference between the Rule of Three and the Inverse Rule of Three?
A. Answers will vary.

7. Joint Ventures
Pre-viewing question
Q. Give two reasons that people might go into business together.
A. Answers will vary.

Post-viewing question
Q. How should Sue and Tanya divide $100 profit if Sue invested $15 for 8 weeks and Tanya invested $10 for 18 weeks?
A. Sue has $15 ×8 weeks or 120 dollar-weeks of equity and Tanya has $10 × 18 weeks or 180 dollar-weeks of equity, so Sue should get 120 / (120 + 180) or 2/5 of the profit and Tanya should get 180 / (120 + 180) or 3/5 of the profit. Sue should get $40 and Tanya should get $60.

8. Tare and Tret
Pre-viewing question
Q. If a crate of 32 oranges weighs a total of 7 pounds, and the crate weighs 1 pound, how much does each orange weigh?
A. (7 pounds – 1 pound) / 32 = 6/32 pounds or 3/16 pounds

Post-viewing question
Q. Explain the meaning of the terms "tare "and "tret."
A. Tare is allowance made to the price of a container of goods for the weight of the container. Tret is extra goods given to a merchant to compensate for what can be expected to be lost in transit.

9. Measuring Molasses
Pre-viewing question
Q. Why is it generally more difficult to measure the volume of containers with curved sides than those with flat sides?
A. Answers will vary.

Post-viewing question
Q. Why was it necessary to take multiple dipstick measurements when estimating the amount of molasses in a crate?
A. Crates varied in shape, so those with different volumes could have one dipstick measurement in common; taking multiple measurements reduced this effect.

10. The Barter System
Pre-viewing question
Q. Why is it easier to use money than to trade goods and services?
A. Answers will vary.

Post-viewing question
Q. In a market with eight goods, all of which are occasionally traded for each other, how many prices must exist?
A. 8 (8 – 1) / 2 = 28 prices, because every good must have a price in relation to every other good.

11. Exchange Rates
Pre-viewing question
Q. What is the difference between American and Canadian dollars?
A. Answers will vary.

Post-viewing question
Q. Explain how an exchange rate is used to convert money from one currency to another.
A. Answers will vary.

12. Credit
Pre-viewing question
Q. Why are banks and businesses willing to offer credit?
A. Answers may include that the banks profit from the interest they receive.

Post-viewing question
Q. Name two benefits of credit for the consumer.
A. Answers will vary.

13. Compound Interest
Pre-viewing question
Q. What is interest?
A. Interest is money paid in addition to the principal of a loan, or accumulated by the principal in a bank account.

Post-viewing question
Q. Explain why the compound interest formula $S = P(1+r)^n$ is equivalent to the compound

interest formula $I = P(1+\frac{r}{n})^{nt}$.

A. S and I both indicate the end value. In the first formula r is the periodic rate of interest, which is the annual interest rate divided by the number of intervals per year, which are the two quantities indicated by r and n in the second formula. In the first formula, n is the total number of compounding intervals, which equals the product of the number of years and the number of intervals per year, which are the quantities represented by n and t in the second equation.

13. Future Value
Pre-viewing question
Q. In banking, what is principal?

A. Principal is the initial amount of money involved in a loan or bank account; it can accumulate interest.

Post-viewing question
Q. A one-year loan accumulates compound interest at a rate of 12% per year and is compounded every month. If the interest rate doubles to 24%, would the amount of interest accumulated after one year double, more than double, or less than double? Explain.
A. The interest rate per compounding interval, r, would change from .01 to .02. The quantity $(1 + r)$ would change from 1.01 to 1.02, and $(1+r)^n$ would change from 1.127 to 1.268. 26.8% is more than twice 12.7%, so the amount of interest owed would more than double.

14. Present Value
Pre-viewing question
Q. For simple interest, how would you find the interest rate if you knew the interest, the principal, and the time in years?
A. The interest rate would equal the interest divided by the product of the principal and the time in years.

Post-viewing question
Q. State the compound interest formula that gives present value P in terms of the end value of the loan S, the number of compounding intervals n, and the interest rate per interval r.
A. $P = \dfrac{S}{(1+r)^n}$

15. Compounding Intervals
TRT: To Come
Pre-viewing question
Q. How many times is interest added to a six-year loan compounded quarterly, or four times per year?
A. 24 times

Post-viewing question
Q. If the yearly interest rate, the principal, and the length of the loan are constant, why will more interest accumulate if the compounding intervals are shorter?
A. Answers may include that interest is reevaluated more frequently, so interest begins accumulating more interest more often.

16. Annual Percentage Rate
Pre-viewing question
Q. What is an interest rate?
A. An interest rate is a percentage determining the amount of the principal that is added to a loan or bank account after a certain period of time.

Post-viewing question
Q. What is the annual percentage rate?
A. The annual percentage rate is the yearly percent increase in a loan after the effects of compound interest are taken into account.

17. The number *e*
Pre-viewing question
Q. What is an irrational number?
A. An irrational number is one that cannot be expressed as the quotient of two integers.

Post-viewing question
Q. What is the value of *e* to three decimal places?
A. 2.718

18. Using *e* to Find Compound Interest
Pre-viewing question
Q. Why are approximations useful?
A. Answers will vary.

Post-viewing question
Q. If you invest $100 at a yearly interest rate of 20% three years, and the compounding intervals are very frequent, give an approximation for the value of the investment at the end of the three years.

A. The formula $S = Pe^{rt}$ gives the end value when the interest is compounded continuously, and approximates end value closely when the compounding intervals are frequent. Here $P = \$100$, $r = .20$, and $t = 3$. The end value is approximately $18